### Case Closed - Evidence:

Twenty players participated in the golf tournament. The average (mean) score was 74.85, or about 75. This is 3 points above par. As the histogram shows, seven players scored at par or below, seven slightly above par (73-78), and the remaining six were well above par. The median score was 75.5, which means that half of the players scored below 75.5 and half scored above 75.5. The highest score was 83 and the lowest was 67 for a range of 16 points. This wide range suggests that there was much variation among the players' performances in this tournament.

### Classroom Cases:

1. Given the data below, organize it in a stem-and-leaf plot, frequency table, or line plot.

Scores in a local golf tournament: 81, 82, 76, 79, 68, 70, 80, 67, 76, 75, 82, 67, 67, 77, 73, 72, 72, 74, 76, 83

#### Case Closed - Evidence:

<table>
<thead>
<tr>
<th>Golf Scores</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 7 7 7 7 8</td>
</tr>
<tr>
<td>7 0 2 2 3 4 5 6 6 6 7 9</td>
</tr>
<tr>
<td>8 0 1 2 2 3</td>
</tr>
</tbody>
</table>

Key: 6|7 = 67

#### Case Closed - Evidence:

2. Display the data in an appropriate graph and discuss the outcome of the tournament.

#### Case Closed - Evidence:

Twenty players participated in the golf tournament. The average (mean) score was 74.85, or about 75. This is 3 points above par. As the histogram shows, seven players scored at par or below, seven slightly above par (73-78), and the remaining six were well above par. The median score was 75.5, which means that half of the players scored below 75.5 and half scored above 75.5. The highest score was 83 and the lowest was 67 for a range of 16 points. This wide range suggests that there was much variation among the players' performances in this tournament.

3. Suppose we want to describe the typical middle school teacher. We are going to conduct a survey among a sample of teachers across the state. Please list five questions we might pose to gather responses that will help with our description.

#### Case Closed - Evidence:

1. What subject(s) do you teach?
2. How many years have you been teaching?
3. How much time do you spend planning lessons?
4. What responsibilities do you have other than teaching, such as club sponsor, coach, etc.?
5. Do you have any hobbies? If so, please list two.
Further investigations:
Here are some activities you can do with your student.

Items that are used together (like hot dogs and buns or paper cups and paper plates) are often sold in differently-sized packages. Look for examples of these in stores. Discuss the smallest number of packages of each item you must buy so that every hotdog has a bun or plates and cups match exactly. The total number of each type of item (hotdog, bun, plate, cup) will be the LCM of their package sizes.

Look at the numbers on car license plates. Discuss whether the number is a prime or a composite and explain how you can tell. If it is composite, find its prime factorization.

Consider the house numbers of houses on your street. Are any of them square numbers?

**Terminology:**

**Multiple:** The product of a given number and a whole number.

**LCM:** The smallest number that is a multiple of two or more numbers.

**Factor:** A whole number that divides evenly into another whole number. Also the process of identifying the divisors (factors) of a given number or expression.

**GCF:** The largest number that is a factor of two or more numbers.

Decompose: To factor numbers or expressions.

**Prime:** A number whose only factors are itself and the number one. (The number one is neither prime nor composite.)

**Composite:** A number which has more than two factors.

**Square Number:** A number that is the product of another number multiplied by itself.

**Book’em:**

**My Full Moon is Square** by Elinor J. Pinczes

**The Doorbell Rang** by Pat Hutchins

**Spaghetti and Meatballs for All** by Marilyn Burns

**Related Files:**

www.ceismc.gatech.edu/psi

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**Fun and Games – Number Theory**

**Students will:**

- Calculate multiples and factors of given numbers
- Identify prime, composite, and square numbers
- Decompose numbers into their prime factorizations
- Determine the Least Common Multiple (LCM) and the Greatest Common Divisor (GCD) for a set of numbers

**Classroom Cases:**

1. There are two traffic signals downtown. One signal light flashes north every four seconds. The other signal light flashes north every five seconds. If they both flash north at 8PM, in how many seconds will they again both flash north?

   **Case Closed - Evidence:**
   One signal flashes north in multiples of four seconds and the other signal flashes north in multiples of five seconds. So I have to find the least common multiple of 4 and 5:
   - 4: 4, 8, 12, 16, 20, 24, 28, 32
   - 5: 5, 10, 15, 20, 25, 30
   - The least common multiple of 4 and 5 is 20. They will both flash north again in 20 seconds!

2. Use the clues below to determine my secret number.

   **Clue 1:** My number is a factor of 72.
   **Case Closed - Evidence:** 72 is a multiple of 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, and 72.

   **Clue 2:** 48 is a multiple of my secret number.
   **Case Closed - Evidence:** 48 is a multiple of 1, 2, 3, 4, 6, 8, 12, 24, and 48.

   **Clue 3:** My number is prime.
   **Case Closed - Evidence:** There are only two prime numbers in the list above: 2 and 3.

   **Clue 4:** My number is even.
   **Case Closed - Evidence:** The secret number is 2.

3. Brandon won the School Box Sweepstakes. He received 288 pencils and 120 notebooks. He decided to share his winnings equally among his friends. Everyone will receive the same number of pencils and everyone will get the same number of notebooks. What is the greatest number of friends Brandon could have and how many notebooks and pencils will each friend get?

   **Case Closed - Evidence:**
   Since the friends who receive pencils are the same people as those who receive notebooks, I need to find one value that will represent them. That value must divide evenly into 288 and 120. Here are the factors (divisors) of each number:
   - 120: 1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 30, 40, 60, 120
   - 288: 1, 2, 3, 4, 6, 8, 12, 24, 36, 48, 72, 96, 144, 288
   - The common factors represent the possible number of friends Brandon has. For these numbers of friends, here is a distribution of notebooks and pencils:

<table>
<thead>
<tr>
<th>No. of friends</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>12</th>
<th>24</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of notebooks</td>
<td>120</td>
<td>60</td>
<td>40</td>
<td>30</td>
<td>20</td>
<td>15</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>No. of pencils</td>
<td>288</td>
<td>144</td>
<td>96</td>
<td>72</td>
<td>48</td>
<td>36</td>
<td>24</td>
<td>12</td>
</tr>
</tbody>
</table>

   - The greatest number of friends Brandon could have is 24. Brandon could give each friend 5 notebooks and 12 pencils.
Further investigations:
Here are some activities you and your student can do together:
Convert batting averages to percents and fractions in lowest terms.
List five items on a grocery receipt in order from least to greatest. List five other items in order from greatest to least. Calculate the tax paid on your purchases.
Practice halving, doubling and tripling the amount of ingredients in recipes that contain fractional measures.
Read ads and calculate prices for items on sale for 10%, 25%, or 50% off.
Calculate tips at restaurants and taxes for items purchased at retail stores.
Calculate the fraction, decimal, and percent of wins for your favorite team.

Terminology:
Fraction: A number that can be written as a quotient of two quantities.
Decimal: A fraction in which the denominator is a power of ten.
Denominator: The number below the line in a fraction. The denominator indicates what kind or size of parts the numerator counts.
Numerator: The term above the line in a fraction. The numerator tells how many parts are being talked about or considered.
Percent: A fraction or ratio in which the denominator is 100.
Proportion: An equation which states that two ratios are equal.
Ratio: A comparison of two quantities that have the same unit of measure.
Rational number: A number that can be written as a/b where a and b are integers, but b is not equal to zero.

Fractions, Decimals, Ratios and Percents
Students will:
• Use fractions, decimals, and percents interchangeably
• Order and compare rational numbers
• Operate with fractions, decimals, and percents
• Use ratios to compare quantities and solve problems

Classroom Cases:
1. Perform the indicated operations. Write your answers in simplified fractional form and then in equivalent decimal and percent forms.
   a. Add: 3 ¾ + 1 ¼
   b. Subtract: 4 ¾ - 3 3/8
   c. Multiply: 5 ½ × 2 ¼
   d. Divide: ½ ÷ ¾

Case Closed - Evidence:
   a. 4 ¾ = 4.875 = 487.5%
   b. 1 ¾ = 1.375 = 137.5%
   c. 12/12 = 1200%
   d. ¾ = 1.2 = 120%

2. Jamil completed 82% of the problems assigned. Patrice finished ⅞ of the problems. Lamar did 17 out of 20 problems. Who did the most if they were all working on the same assignment?

Case Closed - Evidence:
Since ⅞ = 87.5% and 17 out of 20 is equivalent to ⁵⁄₂₀ which is equal to 85%, Patrice did the most problems.

3. Write a math expression you would use to represent each situation.
   a. What is the area of a rectangular plot that is ¾ mile long and ½ mile wide?
   b. How many glasses of water can you pour from 4/5 of a jug if 1/10 of a jug will fill 1 glass?

Case Closed - Evidence:
   a. ⁴⁄₅ × ½
   b. ⁴⁄₅ ÷ ¹⁄₁₀

4. You treated your mother to lunch on her birthday. You paid $ 5.45 for her lunch and $4.85 for your lunch. If you left a 15% tip, how much did you pay altogether?

Case Closed - Evidence:
$11.85 = 1.15 × (5.45 + 4.85)

5. In a survey on favorite fruits, 18 people chose apples, 12 chose oranges, and 10 picked pears. What percent preferred pears?

Case Closed - Evidence:
10/(18+12+10) = ¹⁰/₄₀ = ¼ = 0.25 = 25%

6. Roger’s mother baked banana bread. His sister took ¼ of the loaf to eat with her lunch. Roger’s mother left a note for Roger saying that he could eat ⅓ of what was left. Roger did what the note said. How much of the loaf of banana bread was left after Roger ate his portion?

Case Closed - Evidence:
1 loaf – ¼ loaf = ¾ loaf remaining after sister’s lunch
⅓ × ¾ = ¼ loaf - amount Roger ate
1 loaf - ¼ loaf - ¼ loaf = ½ loaf left

Book’em:
Dad’s Diet by Barbara Comer
One Riddle, One Answer by Laura Thompson

Related Files:
www.ceismc.gatech.edu/csi

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Early Algebra

Students will:

- Use letters to represent numbers
- Write and evaluate algebraic expressions, including those with exponents
- Generalize patterns by writing simple equations using two variables
- Solve simple one step equations using each of the four basic operations.

Classroom Cases:

1. Kayla has five packages of pencils. She gave six pencils to a friend.
   Let $n$ = the number of pencils in one package.
   - Write an expression to show how many pencils Kayla has.
   - Each package contains 12 pencils. Use substitution to write a numerical expression for the number of pencils Kayla has.
   - How many pencils does Kayla have?
   Case Closed - Evidence: a. $5n - 6$  b. $5 	imes 12 - 6$  c. 54

2. Deshawn is saving money to buy a bike. He received $15 for his birthday and deposited it in his bank. He earns money each week for mowing grass or raking leaves. The table below shows how much Deshawn has saved in four weeks.

<table>
<thead>
<tr>
<th>Weeks</th>
<th>Amount Saved</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$15</td>
</tr>
<tr>
<td>1</td>
<td>$27</td>
</tr>
<tr>
<td>2</td>
<td>$39</td>
</tr>
<tr>
<td>3</td>
<td>$51</td>
</tr>
<tr>
<td>4</td>
<td>$63</td>
</tr>
</tbody>
</table>

   - How much money did Deshawn earn each week?
   - How much will Deshawn have saved at the end of 10 weeks? Explain how you know.
   - How many weeks will Deshawn have to save his earnings in order to buy the bike if the bike costs $230?
   Case Closed - Evidence: a. The difference from week to week is $12 as seen below:
   - $27 - 15 = 12$
   - $39 - 27 = 12$
   - $51 - 39 = 12$
   - $63 - 51 = 12$
   - Deshawn earned $12 each week.
   - b. At the end of 10 weeks, Deshawn will have saved $135. I know this because he started with $15 and added $12 	imes 10$ or $120$. $15 + 12 	imes 10 = 135$.
   - c. $A = 15 + 12n$
   - d. Deshawn started with $15. So he has to save $230 - 15 = $215. Since he saves $12 each week, he must divide $215 by $12$ to find the number of weeks to save:
   - $215 + 12 = 17.92 = 18$ weeks.
   - How much will Deshawn have saved after $n$ weeks?
   - How many weeks will Deshawn have to save his earnings in order to buy the bike if the bike costs $230?
   Case Closed - Evidence: a. How much money did Deshawn earn each week?
   - b. How much money did Deshawn have saved at the end of 10 weeks? Explain how you know.
   - c. Let $n$ = the number of weeks. Let $A = 15$.
   - How many weeks will Deshawn have to save his earnings in order to buy the bike if the bike costs $230?
   Solve algebraically and show your work.

Case Closed - Evidence:

3. Evaluate each expression if $a = 6$ and $b = 8$:
   - $a + b$
   - $ab$
   - $3a - 2b$
   - $\frac{a}{b}$
   - $ab^2$
   - $\frac{2b}{a}$

Case Closed - Evidence:

4. Solve for $x$:
   - a. $7x = 98$
   - b. $27 + x = 59$
   - c. $8 = \frac{x}{2}$
   - d. $11 = x - 6$

Case Closed - Evidence:

5. Solve for $x$:
   - a. $x = 14$
   - b. $x = 32$
   - c. $x = 16$
   - d. $x = 17$

Book’em:

Pattern by Henry Pluckrose
Number Patterns Make Sense by Howard Fehr
A Game of Functions by Robert Froman

Related Files:
www.ceismc.gatech.edu/csi

Further investigations:
Try these activities with your student:

Look for and share situations that can be represented with algebraic expressions.
For example, if hotdogs are sold in packs of 8, an expression that shows the number of hotdogs in $h$ packs would be $8h$.

Write an expression for finding the daily temperature change (high temp. – low temp.). Evaluate your expression each day using temperatures reported on the Weather Channel or in the newspaper.

Sort items (laundry, tools, silverware, etc.) around the house making sure only items that are alike are grouped together-
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Further investigations:
Try these activities with your student.

Use a ruler to measure the diameters of cans in the pantry. Determine the radius of each can and then calculate the area for the base of each can. Check your work by drawing a square with approximately the same area as the base of one of your cans. Place the can over the drawing. Does the base of the can just about fit the drawing of the square?

Use a protractor to measure the angles of plane figures around the house. Draw some angles and measure them. Create a picture with your measured angles.

Find circle graphs and frequency tables in newspapers and magazines and discuss what they show and how they are useful.

Terminology:

Central Angle: An angle that has its vertex at the center of a circle.

Chord: A line segment that joins two points on the circumference of a circle.

Circle: The set of all points in a plane that are the same distance (radius) from a given point (center).

Circle Graph: A diagram that displays data in the form of a circle. The circular region is divided into a number of sectors to represent portions of the data. Also called a pie chart.

Circumference: The distance around a circle.

Diameter: The distance across a circle through its center. The line segment that includes the center and whose endpoints lie on the circumference of the circle.

Frequency Table: A table for organizing a set of data that shows the number of times each item or value appears.

Pi (π): The ratio of the circumference of a circle to the diameter of that same circle (π = C/d).

Protractor: A tool for measuring angles.

Radius: The distance from the center of a circle to any point on the circumference. Also, the line segment that has the center of the circle as one endpoint and a point on the circle as the other endpoint.

Sector: A pie-shaped portion of a circular region formed by two radii and an arc of the circle whose endpoints lie on that circle.

Circles and Circle Graphs

Students will: Sixth Grade 5 of 10

- Use circle relationships to find radius, diameter, circumference and/or area of a circle given appropriate information
- Organize data in grouped frequency tables and use the tables to create circle graphs
- Operate with fractions, decimals, and percents to answer questions related to graphs.
- Evaluate algebraic expressions and solve algebraic equations related to circles

Classroom Cases:
1. The diameter of a circular lid is 10 cm. What is the length of the radius of the lid? About how many centimeters is the circumference of the lid? What is the area of the lid to the nearest tenth of a cm²?

Case Closed - Evidence:
Radius is ½ of diameter or 5cm.
Circumference is π × diameter or about 3.14 × 10 = 31.4 cm.
Area is π × r × r or about 3.14 × 5 × 5 = 78.5 cm²

2. 200 people were surveyed about their favorite music. Use the circle graph below to answer the following questions:

- a. Which type of music is the biggest favorite? What percent of those surveyed chose this type?
- b. Of the 200 people were surveyed how many said rock music?
- c. What fraction of the people surveyed chose classical or gospel?
- d. How many degrees are in the central angle for the sector representing oldies music?

Case Closed - Evidence:

- a. country 50%-10% = 40%  b. 0.35 x 200 = 70 people
- c. 15% = 15/100 = 3/20  d. 10% × 360° = 0.10 × 360° = 36°

3. A formula for finding the area of a circle is $A = \pi r^2$. Explain how to use this formula to find the radius of the circle if you know the circle’s area.

Case Closed - Evidence:
To find the radius means to solve the equation for $r$ or to get $r$ alone on one side of the equal sign (=) by using inverse operations. First I would divide both sides of the equation by $\pi$:

$A = \pi r^2$

Then I would think of what number multiplied by itself would give me $A/\pi$, approximately.

Book’em:
The Joy of Pi by David Blatner

Sir Cumference & the Dragon of Pi
and Sir Cumference & the First Round Table
by Cindy Neuschwander

Related Files:
www.ceismc.gatech.edu/csi
Further investigations:
Try these activities with your student:
Look for patterns in home furnishings (rugs, floor tiles, and wallpaper) that are symmetrical.
Discuss the type of symmetry and identify the lines of symmetry and/or the degree of rotation. Write your name and your child’s name in capital letters. Together, examine each letter and draw all lines of symmetry. Are there some letters that have both reflectional and rotational symmetry? Are there some letters that have neither type of symmetry?
Look for symmetrical shapes in nature (leaves, trees, pinecones). Look for asymmetrical shapes as well. Can you find a purpose or advantage in symmetrical properties?
Stand in front of your house. Is it symmetrical? If not, have your student make a sketch and add what might be needed to make the house have symmetry.
Examine hubcaps on several cars. Do they have rotational symmetry? Why? Can you determine the degree of rotation?

Terminology:
Asymmetrical: Describes any figure that cannot be divided into two parts that are mirror images of each other. Asymmetrical means “not symmetrical.”
Axis of or line of symmetry: A line that a figure can be folded over so that one-half of the figure matches the other half perfectly; a line about which a figure is symmetrical.
Line symmetry: Figures that match exactly when folded in half have line symmetry.
Reflectional symmetry: A figure has reflectional symmetry if, after reflecting the figure over a line, the figure lines back up with itself.
Rotation: A transformation that turns a figure about a fixed point at a given angle and in a given direction.
Rotational symmetry: A figure has rotational symmetry if, after rotating it by an angle of 180 degrees or fewer about its center, the figure lines back up with itself.
Symmetry: The property of a figure or expression that allows for parts of it to be interchanged without forcing a change in the whole.

Symmetry

Students will:
- Determine and use lines of symmetry
- Investigate and use rotational symmetry
- Identify objects that have symmetrical properties

Classroom Cases:
1. Do the figures below have lines of symmetry? If so, draw those lines.

   a.  
   
   b.  

   Case Closed - Evidence:
   a. There are two lines of symmetry:

   b. This figure does not have any lines of symmetry.

2. Do the figures above have rotational symmetry? If so, what is the degree of rotation?

   Case Closed - Evidence:
   a. This figure has rotational symmetry. If it is rotated 180° around its center point, it lines up with itself.
   b. This figure has rotational symmetry. If it is rotated 72°, it will look just like itself.

3. Sketch a symmetrical object from your home and describe its properties of symmetry.

   Case Closed - Evidence:
   At right is my stovetop. It has two lines of symmetry (dashed). It has rotational symmetry. When it is rotated 180° around its center point, the stovetop will look the same.

Book’em:
Islamic Designs in Color by N. Simakoff
Natural Art Forms by Karl Blossfeldt
Kaleidoscope Design Coloring Book by Lester Kubistal
Snow Crystals by W.A. Bentley
Art Forms in Nature by Ernst Heinrich Haeckel

Related Files:  
www.ceismc.gatech.edu/csi
Further investigations:
Here are some activities you and your student can do together:
Practice measuring large and small items using both metric and customary measurement. For example, measure the dimensions of a room or the dimensions of a book. Choose two rectangular prisms (such as cereal and cracker boxes). Predict which box has the greater volume. Then measure length, width, and height to find the volumes ($V = l \times w \times h$).

Calculate distance to nearby cities using a map and its scale.

Compare a toy such as a car or a dollhouse to the actual object it represents. Is the toy a scale model? If so, what is its scale factor?

Terminology:
Dimensions: The measure of the magnitude, or size, of an object.
Proportion: An equation which states that two ratios are equal.
Ratio: A comparison of two quantities that have the same unit of measure.
Scale drawings: Drawings that represent relative sizes and placements of real objects or places.
Scale factor: The ratio of lengths of corresponding sides of two similar figures.
Similar figures: Figures that have the same shape but not necessarily the same size.
Unit: A fixed amount that is used as a standard of measurement.

Scale Factor

Students will:

- Select and use appropriate units to measure length, perimeter, area, and volume
- Measure lengths to the nearest $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, and $\frac{1}{16}$ inch
- Convert one unit of measure to another in the same system of measurement
- Use ratio, proportion, and scale factor to describe relationships between similar figures
- Interpret and create scale drawings
- Solve problems using scale factors, ratios, and proportions

Classroom Cases:

1. Amir is 1.6 meters tall. How tall is he in centimeters?
   - Amir is 1.6 meters tall. How tall is he in centimeters?
   - $1.6 \text{ m} = 160 \text{ cm}$

2. How long is the pencil?
   - Measure to the nearest $\frac{1}{2}$ inch.
   - Then measure to the nearest $\frac{1}{8}$ inch.

   - There are 16 divisions (little lines) in each inch. The eraser of the pencil is aligned with 0 and the point of the pencil is aligned with the 7th line past the 4 inch mark. So the pencil measures $4 \frac{1}{4}$ inches. To the nearest $\frac{1}{2}$ inch, $4 \frac{1}{4}$ inches is between $4 \frac{1}{4}$ and $4 \frac{1}{2}$ inches. $4 \frac{1}{4}$ inches is closer to $4 \frac{1}{2}$ inches than to $4$ inches.
   - $4 \frac{1}{4} \text{ in.}$

3. Anna's living room floor is a rectangle 12 feet by 15 feet. How many square feet of carpeting will she need to cover the floor?
   - The dimensions of the scale drawing are $1 \frac{1}{4}$ foot by $\frac{1}{4}$ inch. I find the actual measurements:
     - $12 \text{ feet} = x \text{ feet}$
     - $x = 1 \frac{1}{4} \times 12 = \frac{5}{4} \times 12 = 15 \text{ feet}$
     - $12 \text{ feet} = y \text{ feet}$
     - $y = \frac{7}{8} \times 12 = 8 \frac{1}{2} \text{ feet}$
     - $12 \text{ feet} = z \text{ feet}$
     - $z = \frac{11}{16} \times 12 = 10 \frac{8}{16} = 10 \frac{1}{2}$ feet

   - The volume of the living room is $xyz = 15 \times 10 \frac{1}{2} \times 6 \frac{3}{4} = 1063.13 \text{ cubic feet}$
Further investigations:
Cut open a cereal or cracker box to form a flat shape with six rectangles. Find the area of each rectangle and total the areas to find the box’s surface area.

Cut open an empty frozen orange juice can to make a flat shape with two circles (lids) and a rectangle. Find areas of each flat shape and add results to find the total surface area.

Use a ruler to measure the diameters and heights of cans in the pantry. Calculate the volumes and surface areas of the cans using formulas.

Use a ruler to measure the length, width, and height of boxes or box-shaped objects such as a desk, a TV, or a refrigerator. Calculate the volumes and surface areas of the boxes and other rectangular prisms.

Ask your student to show you how to fold nets (patterns) to make solid figures.

Terminology:
Base: Flat circular part of a cone or polygonal region of a pyramid that does not intersect with other faces at the apex.
Bases: Two parallel and congruent regions of a prism or cylinder. Bases are circular in a cylinder and polygonal in a prism.
Cone: A 3-D figure with one circular or elliptical base and one vertex.
Cube: A regular polyhedron whose six faces are congruent squares.
Cylinder: A 3-D object with two parallel, congruent circular bases.
Edge: The intersection of a pair of faces in a 3-D object.
Face: one of the sides of a polyhedron.
Net: A 2-D figure that, when folded, forms the surfaces of a 3-dimensional object.
Polyhedron: A 3-D figure that has polygons as faces.
Polygon: A closed figure formed by three or more line segments.
Prism: A polyhedron with two parallel and congruent faces called bases, and other faces that are parallelograms.
Pyramid: A polyhedron with one base and the same number of triangular faces as there are sides of the base.
Surface area: Total area of the 2-D surfaces that make up a 3-D object.
Volume: The amount of space occupied by an object.

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Solids
Students will:

- Name and compare properties of simple geometric solids
- Estimate volumes and surface areas of basic solid figures
- Apply formulas to compute volumes and surface areas of solids
- Interpret and sketch various views of solids
- Construct nets for prisms, cylinders, pyramids, and cones

Classroom Cases:
1. For each figure below, state its mathematical name, estimate its volume and its surface area, then use formulas to compute volume and surface area.

   ![Figure 1](image1)

   - **A.** This is a prism. Its estimated volume is 5×6×8 = 240 ft³ and its estimated surface area is 2×40 + 2×48 + 2×30 = 2 (40+30+48) = 237 ft²
   - **B.** This is a cylinder. Its estimated volume is 3×4×4×6=288 m³, and its estimated surface area is 2×3×4×4 + 3×8×6 = 240 m²
   - **C.** This is a cone. Its estimated volume is 28 in³.
   - **D.** This is a pyramid. Its estimated volume is 4 cm³.

2. For each figure below, state its mathematical name, estimate its volume, and then use formulas to compute its volume.

   - **A.** This is a cone. Its estimated volume is 28 in³.
   - **B.** This is a cylinder. Its estimated volume is 3×4×4×6=288 m³, and its estimated surface area is 2×3×4×4 + 3×8×6 = 240 m²
   - **C.** This is a cone. Its estimated volume is 28 in³.
   - **D.** This is a pyramid. Its estimated volume is 4 cm³.

3. Draw nets for each of the figures in cases 1 and 2.

4. Sketch the figures described:
   - **A.** A right rectangular prism 2 cm x 3 cm. x 2 cm
   - **B.** A cylinder with diameter of 4 cm and height 3 cm
   - **C.** A pyramid with square base 3 cm x 3 cm and height 4 cm
   - **D.** A cone with radius 2 cm and height 5 cm

Case Closed - Evidence:

- **A.** By formula, \( V = \frac{1}{3} lwh = \frac{1}{3} \times 2 \times 2 \times 3 = 4 \text{ in}^3 \)
- **B.** By formula, \( V = \pi r^2 h = 3.14 \times 4.1^2 \times 5.8 \approx 306.144 \text{ m}^3 \)
- **C.** By formula, \( V = \frac{1}{3} \pi r^2 h = \frac{1}{3} \times 3.14 \times 2.2^2 \times 7 = 35.46 \text{ in}^3 \)
- **D.** By formula, \( V = \pi \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{3} \times 2 \times 2 \times 3 = 4 \text{ in}^3 \)

Case Closed - Evidence:

- **A.** By formula, \( V = \pi r^2 h = 3.14 \times 4.1^2 \times 5.8 = \approx 306.144 \text{ m}^3 \)
- **B.** By formula, \( V = \pi r^2 h = 3.14 \times 4.1 \times 4.1 \times 5.8 \approx 254.9 \text{ ft}^2 \)
- **C.** By formula, \( V = \pi r^2 h = 3.14 \times 4.1^2 \times 5.8 = 306.144 \text{ m}^3 \)
- **D.** By formula, \( V = \pi r^2 h = 3.14 \times 4.1 \times 4.1 \times 5.8 = 254.9 \text{ ft}^2 \)
Further investigations:
Try these activities with your student:
If your student earns an allowance or has a job, make a graph showing how much money is received. Show pay periods on the horizontal axis and total amount received on the vertical axis.
Identify quantities in your environment that vary in the same way. For example, the more movie tickets you buy, the more money you pay. When you enlarge a photograph, how do the dimensions change?
Read ads that contain percent reductions and determine how much sale items will cost solving proportions (see case 2.)
Identify best buys by comparing prices for unit amounts.

Terminology:

- **Constant of proportionality**: The constant value of the ratio of two proportional quantities $x$ and $y$; usually written $y = kx$, where $k$ is the constant of proportionality. Note that $k = y/x$.
- **Direct proportion** (Direct Variation): The relation between two quantities whose ratio remains constant. When one variable increases, the other increases by the same factor. When $A$ changes, then $B$ changes by the same factor: $A = kB$, where $k$ is the constant of variation.
- **Equation**: A mathematical sentence that contains an equals sign.
- **Proportion**: An equation which states that two ratios are equal.
- **Proportional**: Two quantities are proportional if they are multiples of each other.
- **Rule**: A description or an equation that indicates the relationship among variables. An example might be: video games cost $60 each. The variables are number of video games and total cost. We could represent their relationship with an equation: $T = 60n$.

Direct Proportions

**Students will:**

- Draw pictures and use manipulatives to demonstrate a conceptual understanding of proportions
- Solve problems using proportional reasoning
- Represent and recognize direct variation graphically, numerically, and symbolically
- Identify and interpret the constant of proportionality in direct relationships

**Classroom Cases:**

1. Karen is mixing paint. She mixed three cups of red with five cups of white to make her favorite shade of pink. Her brother, Phil, dumped in another cup of red paint. How much white paint must Karen add to make the mixture return to her favorite shade of pink?

**Case Closed - Evidence:**
To get the desired pink color, we need a white to red ratio of 5:3. So, $\frac{w}{4} = \frac{5}{3}$ and $w = \frac{20}{3} \approx 6.67$.

Karen needs about 6.67 cups of white paint. She has already poured in 5 cups of white so she must add 1.67 cups.

2. At the spring sale, prices were marked 20% off. How much would you pay for an item that regularly costs $10.50?

**Case Closed - Evidence:**
When prices are marked off 20%, you must pay 80% of the regular price ($100\% - 20\% = 80\%$).

\[
\frac{p}{10.50} = \frac{80}{100} \quad \text{and} \quad p = 10.50 \times 80 \div 100 = 8.40
\]

You will pay $8.40 for the item.

3. Consider the table below. Does it represent a proportional relationship? How do you know? Write a rule describing the relationship and then express the relationship with an equation.

<table>
<thead>
<tr>
<th>$x$</th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>8</td>
<td>6</td>
<td>4</td>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

**Case Closed - Evidence:**
The quantities in the table vary proportionately $y/x$ is equal to a constant, 2. Each $y$ value is 2 times the corresponding $x$ value.

As an equation, this relationship is: $y = 2x$

**Book’em:**
- If You Hopped Like a Frog by David M. Schwartz
- Jim and the Bean Stalk by Raymond Briggs
- Roll Thunder, Hear My Cry by Mildred D. Taylor
Further investigations:
Here are some activities you and your student can do together:

Analyze baseball statistics. Predict how likely your favorite player is to get a hit on his next at bat.

Play board games that involve the use of spinners (Life, Clue, etc.). Using ratios, describe the probability of landing on each section of the spinner. Keep a tally of where each person lands on each spin and a count of the total number of times he spins. Determine each player's probability for landing in each section of the spinner. Compare these experimental probabilities with theoretical probabilities.

Terminology:

**Equally likely outcomes:** Two or more possible outcomes of a situation that have the same probability.

**Event:** Any possible outcome of an experiment in probability.

**Experimental Probability:** The ratio of the number of times an outcome occurs to the total number of trials performed.

**Population:** The complete and entire set of items or people to be studied.

**Probability:** A measure of the likelihood of an event. It is the ratio of the number of ways a certain event can occur to the number of possible outcomes.

**Ratio:** A comparison of two quantities that have the same unit of measure.

**Sampling:** Collecting data from a subgroup of a population of interest.

**Simulation:** An experiment that models real life experience.

**Theoretical Probability:** The mathematical calculation that an event will happen in theory.

**Trial:** one run of an experiment.

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### Games of Chance

**Students will:**

- Predict the probability of a given event through trials and/or simulations
- Determine the theoretical probability of a simple event
- Represent probability using a ratio
- Distinguish between theoretical and experimental probabilities
- Explain how experimental probability of an event and theoretical probability of that event are related

**Classroom Cases:**

1. You roll a pair of fair six-sided dice.
   - a. What is the probability that the sum of the numbers on the tops of the dice will be 9?
   - b. What is the probability that the sum will be 15?
   - c. What is the probability that the sum will be less than 15?
   - d. What is the probability that the sum will be a prime number?

**Case Closed - Evidence:**

First, I would make a chart so that I can see how many ways different events can occur:

<table>
<thead>
<tr>
<th>2nd Roll</th>
<th>1st Roll</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2 3 4 5 6</td>
</tr>
<tr>
<td>2</td>
<td>3 4 5 6 7</td>
</tr>
<tr>
<td>3</td>
<td>4 5 6 7 8</td>
</tr>
<tr>
<td>4</td>
<td>5 6 7 8 9</td>
</tr>
<tr>
<td>5</td>
<td>6 7 8 9 10</td>
</tr>
<tr>
<td>6</td>
<td>7 8 9 10 12</td>
</tr>
</tbody>
</table>

2. Suppose you picked marbles from a bag. Out of 100 trials, you picked a red marble 30 times. What is the chance of picking a red marble on your next pick?

**Case Closed - Evidence:**

Since I picked a red marble 30 out of 100 times, the experimental probability of picking a red marble would be 30/100 or 0.3. The chance of picking a red marble on the next pick is 3/10. (Notice that you cannot calculate the theoretical probability because you do not know how many marbles are in the bag or how many of them are red.)

3. A local restaurant collects business cards in a fish bowl and once a week draws one to award its owner a free meal. This week there are more than three cards in the bowl; 1/3 are from engineers, 2/5 are from health care professionals, and the rest are from people who work at the courthouse. What is the probability that a courthouse worker wins?

**Case Closed - Evidence:**

Let $c = \text{the fraction of cards from courthouse workers.}$

$\frac{1}{3} + \frac{2}{5} + c = 1$ where $1$ represents all of the cards in the bowl

$5/15 + 6/15 + c = 1$

$5 + 6 + 15c = 15$

$15c = 4$

$c = 4/15$ So a courthouse worker has 4/15 chance of winning this week.

**Book’em:**

- Jumanji by Chris Van Allsburg
- The Phantom Tollbooth by Norton Juster
- Cloudy with a Chance of Meatballs by Judi Barrett
- Math Fun, Test Your Luck by Rose Wyler and Mary Etting
- Do You Wanna Bet? Your Chance to Find Out About Probability by Jean Cushman

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